Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XV (2016), 85-116

Bubble tower solutions for a supercritical elliptic problem in $\mathbb{R}^N$

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Abstract. We consider the problem

$$\begin{cases} -\Delta u + u = u^p + \lambda u^q & u > 0 \text{ in } \mathbb{R}^N \\ u(z) \to 0 & \text{as } |z| \to \infty \end{cases}$$

where $p = p^* + \varepsilon$, with $p^* = \frac{N+2}{N-2}$, while $1 < q < \frac{N+2}{N-2}$ if $N \geq 4$, and $3 < q < 5$ if $N = 3$, $\lambda > 0$, and ε is a positive parameter. We prove that for $\varepsilon > 0$ small enough, the problem has a solution with the shape of a tower of bubbles.

Mathematics Subject Classification (2010): 35J61 (primary); 35B33, 35J08 (secondary).